**Conditional Probability**

**Conditional probability** means finding the chance of an event happening when we already know that another related event has occurred.

It is the probability of an event occurring given that another related event has already occurred.

P(A∣B) = P(A and B) ​/ P(B)

here:

* P(A∣B) is the probability of event A occurring given that event B has occurred.
* P(A and B) is the probability that both events A and B occur.
* P(B) is the probability that event B occurs.

Suppose you have a deck of 52 cards, and you draw a card. What's the probability that the card is a **king**, given that it is **red**?

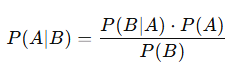
There are 2 red kings out of the 26 red cards

​Probability of King when red has already occurred:

P(King∣Red) = Number of red cards / Number of red kings ​= 2/26​ = 1/13

### ****Bayes' Theorem****

**Bayes' Theorem** is a mathematical formula used to find the **conditional probability** of an event, based on prior knowledge of conditions related to the event.



* P(A∣B) is the probability of event A occurring given that event B has occurred.
* P(B|A) is the probability of event B occurring given that event A has occurred.
* P(A) is the probability that event A occurs.
* P(B) is the probability that event B occurs.

### Example: Medical Test for a Disease

#### **Problem:**

Suppose there is a disease that affects **1% of the population**. There is a medical test for this disease that is **90% accurate**. This means:

Now, you take the test, and the result comes back **positive**. What is the probability that you **actually have the disease**?

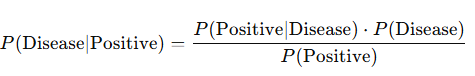
#### What we know:

* The probability that a person has the disease is:  
  P(Disease) = 0.01 (1% of the population has the disease)
* The probability that a person does **not** have the disease is:  
  P(No Disease) = 0.99
* The probability that the test is **positive** given that a person **has the disease** is:  
  P(Positive∣Disease) = 0.90
* The probability that the test is **positive** given that a person **does not have the disease** is:  
  P(Positive∣No Disease) = 0.10

#### We want to find:

The probability that a person **actually has the disease** given that their test result was **positive**, i.e., P(Disease∣Positive).

Bayes' Theorem formula is:



Where:

P(Positive) is the total probability of a positive test result. It includes both people who have the disease and those who do not.



= (0.90 \* 0.01) + (0.10\*0.99)

= 0.108

Now apply Bayes' Theorem:



### Conclusion:

Even though you got a **positive test result**, the probability that you **actually have the disease** is only about **8.33%**.

#### Why is this surprising?

* The **disease is rare** (only 1% of people have it), and the test, while **90% accurate**, still has a **10% false positive rate**.
* Despite a **positive test result**, because the disease is so rare, the probability that you have it is still quite low. This is the power of **Bayes' Theorem**—it helps us understand how prior probabilities (like the rarity of the disease) influence the interpretation of new evidence (like the test result).

Spam detection using Naïve Bayes Classifier underneath it is using Baye’s Theorem

In Naïve Bayes, **Bayes' Theorem** helps update the probability of an email being spam or not, given the presence of certain words.

Probability of the email being spam is proportional to:

* The probability of the email being spam. P(Spam)
* The product of the probabilities of each word occurring in spam emails P(Word|Spam)

The "naïve" part comes from the assumption that the words are independent, which makes the computation simpler.

Thus, Bayes' Theorem provides the foundation for calculating the likelihood of spam classification based on observed words in the email.

What is the probability that an email is spam given that it contains the word “lottery”.

P(spam) = 20%

P(lottery) = 2% (spam or not spam)

P(lottery|spam) = 5% (probability of word ”lottery” in a spam email)

Probability that an email is spam given that it contains the word “lottery”

P(spam|lottery) = [ P(lottery|spam) \* P(spam) ] / P(lottery)

= [ 5 \* 20 ] / 2 = 50%

Bayes' Theorem helps recommendation systems by updating the probability of a user liking an item based on their past behavior (such as ratings, clicks, or purchases). By calculating these probabilities, recommendation systems can make more accurate predictions and suggest items that are likely to align with the user's preferences.

Given that user A has liked a particular item, what’s the probability that User B will also like it?

User B has liked several other items that User A liked.

Language Models – Thanks for Reaching out

Probability of new words given the existing words

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Customer Churn Prediction

Given customer activity in the last n days, what is the probability, they will churn next month?